Micro and Macro Labor Supply Elasticities in a Life Cycle Model With Taxes

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Background: **Taxes and Hours of Work**

Previously we showed that the effect of taxes on hours of work is very dependent on a preference parameter that describes the curvature in the utility from leisure (or disutility from work)

Many economists have estimated this curvature parameter using micro data and concluded that it is very large.

Question: Does this mean that aggregate effects of taxes are small?

Today I will argue that the answer to this question is no.

In particular, I will argue that the estimate of this parameter from micro data is not the appropriate value to use in representative agent aggregate analyses.

Tax/Transfer Programs and Hours of Work-A Simple Model

Single household

Preferences:

$$\log c - \alpha \frac{h^{1+\gamma} - 1}{1 + \gamma}$$

Technology:

$$c = h$$

Government:

$$T = \tau w h$$

Quantitative Assessment of Tax/Transfer Programs

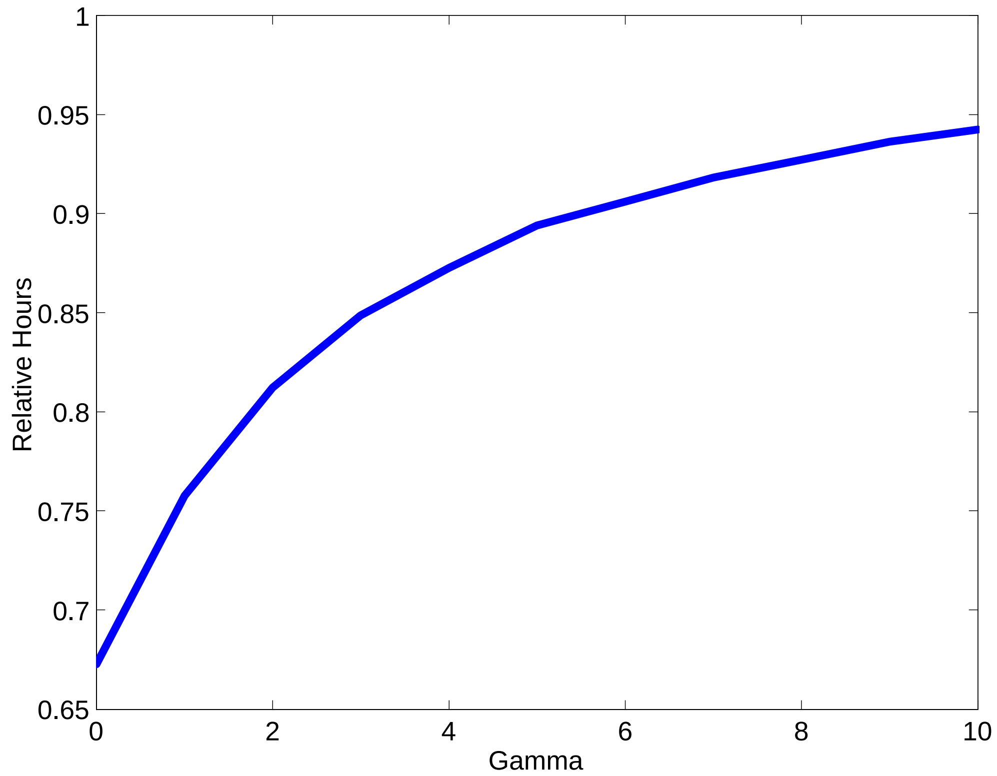
1. Choose a value of γ

2. Calibrate to the US

- Set $\tau = .3$
- Given τ, γ set α so that $h = 1/3$

3. To assess the effect of taxes on hours of work in Europe compute the equilibrium for $\tau = .5$.

Relative hours as a function of γ:



Key Issue:

- What is the appropriate value of γ?

Some additional issues for further work:

- Patterns in hours of work at a disaggregated level
- Explicit modeling of actual tax/transfer programs
- Explaining why countries adopt different tax/transfer programs

What is the appropriate value of γ?

Beginning with MaCurdy (1981), there have been many papers that use individual level data to estimate a utility function of the form:

$$\sum_{a=0}^{T} \beta^t [u(c_t) - \frac{\alpha}{1+\gamma} h_t^{1+\gamma}]$$

First order condition for an interior solution for h_t from the consumer maximization problem is:

$$\beta^t h_t^{\gamma} = \mu \frac{w_t}{(1+r)^t}$$

where μ is the lagrange multiplier on the budget constraint.

Taking logs and rearranging, one gets an equation of the form:

$$\log h_t = A + Bt + \frac{1}{\gamma}\log w_t$$

where

$$B = -\log[\beta(1 + r)]$$

Subject to making some assumptions about randomness, this can be used to generate a regression equation in which the coefficient on $\log w$ provides an estimate of the key preference parameter.

Most of these estimates use data for prime aged males that are continuously employed, and most find that γ is a relatively large number, typically as large as 10.

Many economists proceed as if one can use this estimate of γ in the previous calculation, and conclude that taxes cannot be the dominant source of differences across countries.

In what follows I will present a model and carry out some simulations to show that this is not a good thing to do.

Key features of the model:

The model will be a life cycle model that can account for two key regularities about life cycle labor supply:

(**1**) Positive correlation between hours and wages for prime aged individuals.

(**2**) Endogenous retirement

This will require building a model in which labor supply decisions have both an intensive margin and an extensive margin.

Building Blocks of the Model

Model 1: **Static Indivisible Labor Model**

Continuum of identical households

Preferences: $u(c) + v(1 - h)$, $c \geq 0$, $h \in \{0, \bar{h}\}$

Technology: $C = H$

Optimal allocation: randomly choose a fraction f of workers to be employed but give equal consumption to everyone.

Model 2: **Indivisible Labor Model in Continuous Time**

Preferences: $\int_0^1 [u(c(t)) + v(1 - h(t))]dt$, $c(t) \geq 0$, $h(t) \in \{0, \bar{h}\}$

Technology: $C(t) = H(t)$

Optimal allocation: randomly choose a fraction f of individuals to work at each instant and give everyone constant consumption.

However, one can achieve the same utility without randomization.

In particular, assume the following AD market structure: At time zero there are markets for labor and consumption at all instants of time.

One can show that there is a continuum of equilibrium in which $p(t) = w(t) = 1$ for all t, each individual has $h(t) = \bar{h}$ for a set of measure f, and has constant consumption.

Aggregate quantities are the same in all of the equilibria.

Models with Intensive and Extensive Margins

There are different ways that one can provide foundations for the indivisible labor assumption. The common element is to create a non-convexity somewhere in the economic environment. Assuming fixed costs of work are one possibility.

Prescott, Rogerson, Wallenius (2006) focus on another possibility: non-convexities in the mapping from time at work and labor services provided.

Let h be time devoted to work for an individual, and let l be labor services provided by that individual.

Standard assumption: $l = h$

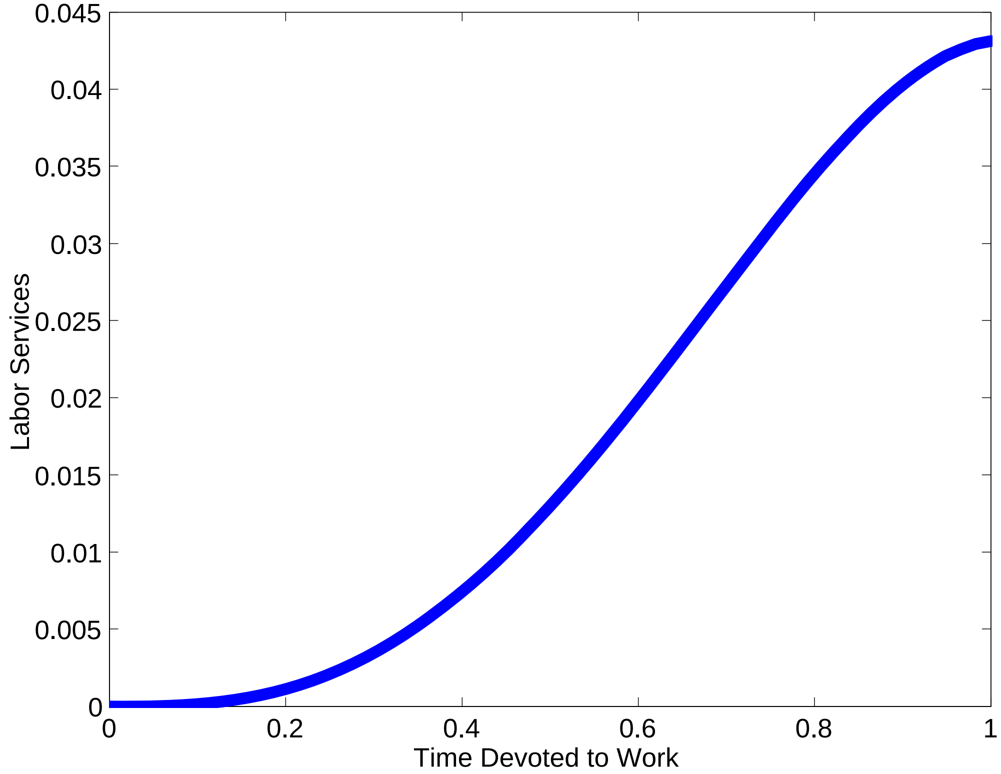
Common extension: Individuals have different productivities and labor input is measured in efficiency units. For an individual with productivity e we have: $l = eh$

Our assumption: The mapping from time to labor services is given by:

$$l = g(h)$$

where g is not concave globally but is concave for $h \geq \bar{h}$.

An example of a $g(h)$ function:



Model 3: **Static Model with Intensive and Extensive Margins**

Unit mass of identical individuals

Preferences: $u(c) + v(1 - h)$

Technology: $C = L$, $l = g(h)$

If the $g(h)$ function has the form shown above then optimal allocations may mimic those with labor indivisibilities, in the sense that it may be optimal to randomize across workers, with some workers working $h^* > 0$ and others working 0, consumption equalized across everyone.

But in this model, the value of h^* is endogenously determined.

Model 4–Model 3 in Continuous Time

If we extend this model to continuous time then we find that optimal allocations can be implemented as equilibria without lotteries, and there is a continuum of equilibria, each of which has the same aggregate outcomes.

This model behaves very much like an indivisible labor model in the sense that in the presence of a tax and transfer scheme like that mentioned earlier, h^* is unaffected by taxes and all of the adjustment comes in the fraction of life spent in employment.

Now we embed the non-linear mapping from time to labor services into a life cycle model and assess consequences of tax and transfer policies.

Model

Continuous time overlapping generations model, length of life normalized to one

Preferences:

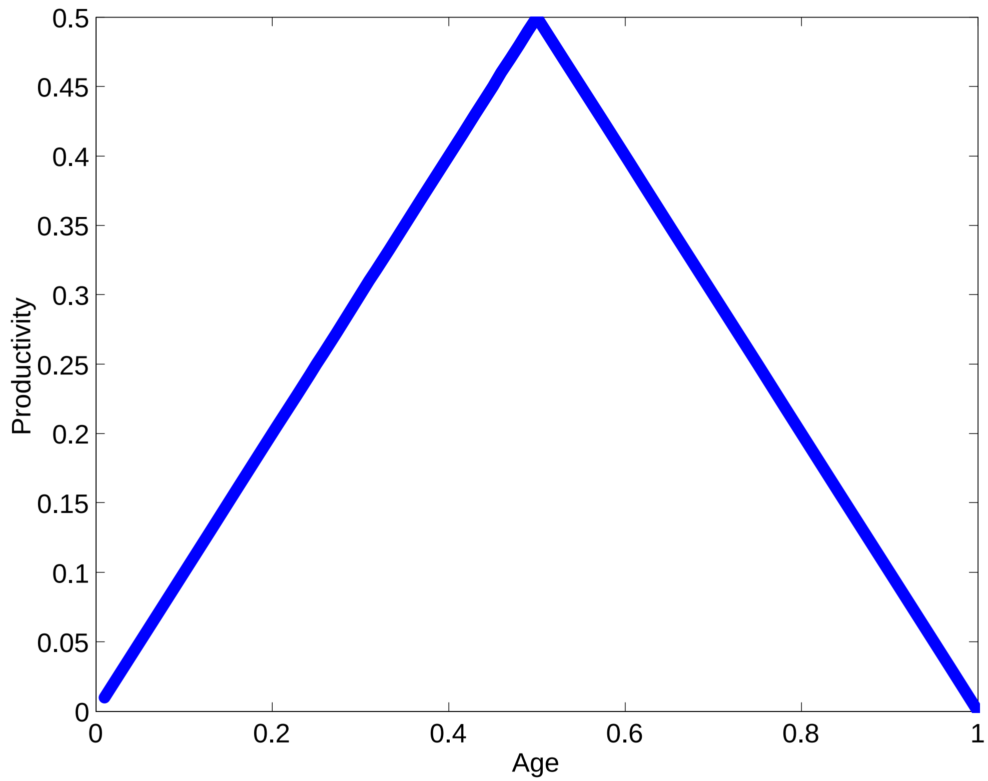
$$\int_0^1 U(c(a), 1 - h(a))da$$

Technology

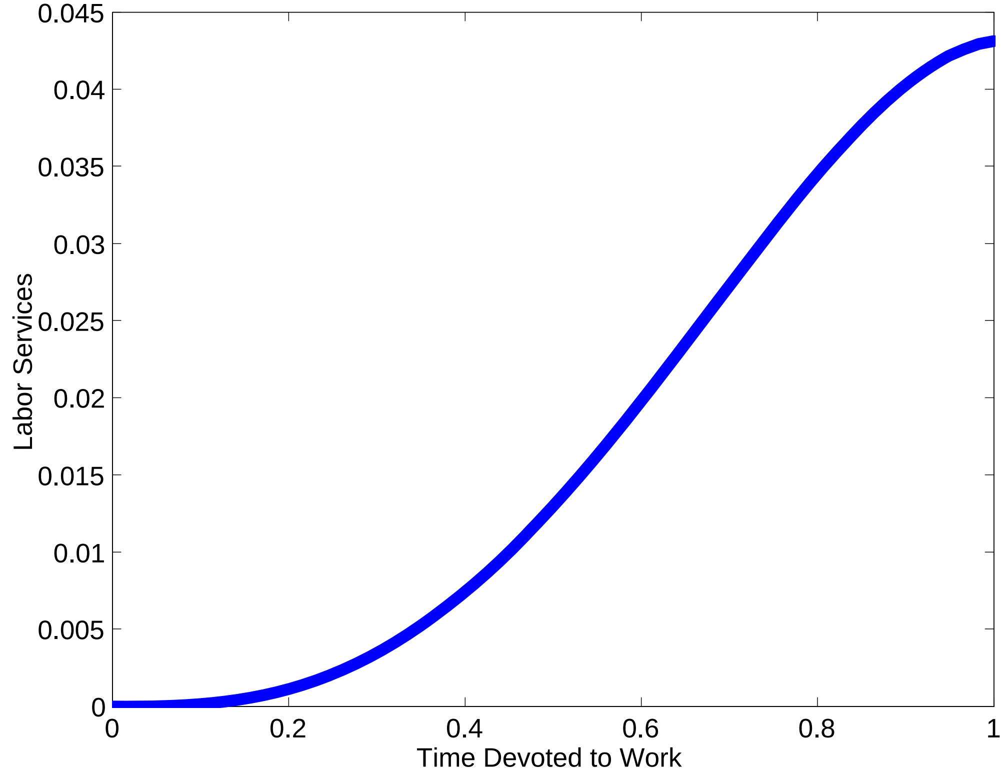
$$Y(t) = L(t)$$

$$l = \tilde{e}(a)g(h)$$

The $\tilde{e}(a)$ function:



The $g(h)$ function:



Equilibrium

We assume there are AD markets for consumption and labor services at each instant.

$p(t) =$ price of a unit of consumption at date t

$w(t) =$ price of a unit of labor services at date t

Profit maximization implies that $w(t) = p(t)$ for all t

We focus on steady state equilibria in which allocations are the same for all generations and $p(t)/p(t+1)$ is constant.

There may be more than one steady state equilibrium. Some of them are not attainable without some supporting government policy such as issuing of debt. For this model there is always a steady state in which prices are constant over time, and we will assume that the government behaves in a way so that this steady state obtains.

Consumer Maximization Problem in Steady State:

Since $p(t)$ is constant, we can normalize it to one, and consumer problem becomes:

$$\max_{c(a),h(a)} \int_0^1 U(c(a), 1 - h(a))da$$

$$s.t. \int_0^1 c(a)da = \int_0^1 \tilde{e}(a)g(h(a))da$$

Result 1: The optimal solution for $h(a)$ has a reservation property. In particular, there exists a value $\tilde{e}^*$ such that $h(a) > 0$ if $\tilde{e}(a) > \tilde{e}^*$ and $h(a) = 0$ if $\tilde{e}(a) < \tilde{e}^*$.

Result 2: Let $h(a)$ be the optimal solution for hours of work over the life cycle. Let a_1 and a_2 be distinct ages for which $h(a) > 0$. Then $\tilde{e}(a_1) > \tilde{e}(a_2)$ implies $h(a_1) > h(a_2)$.

Characterizing Steady State Labor Supply

We restrict preferences to:

$$U(c, 1-h) = \log(c) - v(h)$$

I will show that finding the equilibrium can be reduced to finding two numbers:

1. fraction of life devoted to work
2. hours worked at peak productivity

In steady state, $c(t)$ will be constant. Recalling our earlier result regarding a reservation property for employment, we can rewrite the individual's maximization problem as:

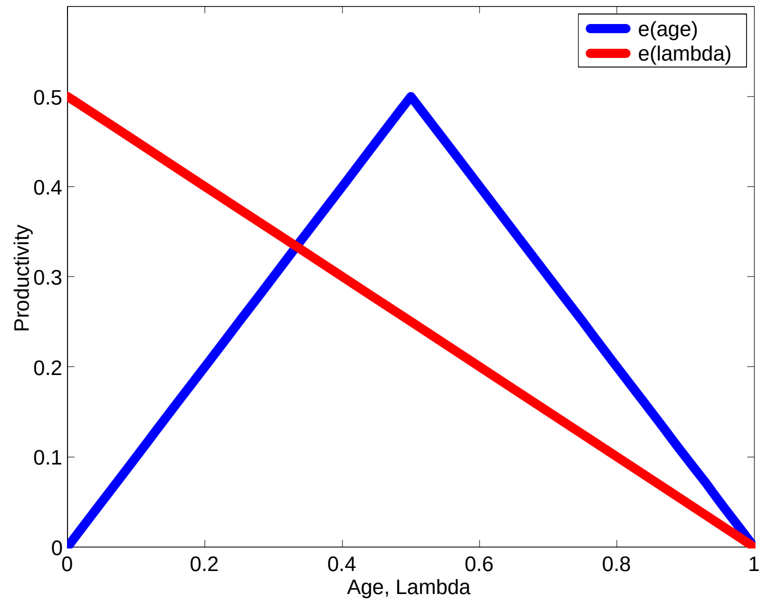
$$\max_{c,h(a),A_1,A_2} \log c - \int_{A_1}^{A_2} v(h(a))da$$

$$s.t.\ c = \int_{A_1}^{A_2} \tilde{e}(a)g(h(a))da$$

Reformulating the Consumer Problem:

Let $e(\lambda)$ be the profile of productivity if we arrange time from highest to lowest productivity, i.e., $e(\lambda)$ solves:

$$\lambda = \int_0^1 I(\tilde{e}(a) \geq e(\lambda))da$$



Consumer problem can be expressed as:

$$\max_{c,h(\lambda),\lambda^*} \log c - \int_0^{\lambda^*} v(h(\lambda))d\lambda$$

$$s.t.\ c = \int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda$$

where λ^* represents the fraction of lifetime spent in employment. Given a value for λ^* it is straightforward to back out the implied values for A_1 and A_2. If solutions for both are interior then they solve:

$$\tilde{e}(A_1) = \tilde{e}(A_2) = e(\lambda^*)$$

First order conditions:

$$v(h(\lambda^*)) \bullet \int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda = e(\lambda^*)g(h(\lambda^*))$$

$$v'(h(\lambda)) \bullet \int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda = e(\lambda)g'(h(\lambda)) \text{ for } 0 \le \lambda \le \lambda^*$$

The second equation implies:

$$\frac{v'(h(\lambda))}{e(\lambda)g'(h(\lambda))} = \text{constant for all } \lambda \in [0, \lambda^*]$$

The left hand side is strictly increasing in $h(\lambda)$, so it follows from this equation that if $h(0)$ is known, the entire $h(\lambda)$ profile is uniquely determined.

Moreover, an increase in $h(0)$ leads to an upward shift of the entire $h(\lambda)$ profile.

It follows that the consumer's problem can be reduced to finding optimal values for $h(0)$ and λ^*.

We show that the equilibrium can be described as the intersection of two curves in $h(0) - \lambda^*$ space, one of which is (at least locally) upward sloping, and the other of which is (globally) downward sloping.

Given that $h(\lambda)$ can be solved in terms of $h(0)$, the equilibrium can be represented as solutions to the following two equations:

$$\frac{v'(h(0))}{e(0)g'(h(0))} = \frac{v(h(\lambda^*))}{e(\lambda^*)g(h(\lambda^*))} \text{ for all } \lambda \in [0, \lambda^*]$$

$$\frac{e(0)g'(h(0))}{v'(h(0))} = \int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda$$

$$\frac{v'(h(0))}{e(0)g'(h(0))} = \frac{v(h(\lambda^*))}{e(\lambda^*)g(h(\lambda^*))} \text{ for all } \lambda \in [0, \lambda^*]$$

Claim: In a neighborhood of the optimal solution, this first equation defines an increasing relationship between $h(0)$ and λ^*.

Proof: To prove this we establish three properties:

- The lefthand side is strictly increasing in $h(0)$.
- Given an optimal profile $h(\lambda)$, the right hand side of equation is increasing in λ.
- The marginal effect of an increase in $h(0)$ on the right hand side evaluated at the optimal λ^* is zero.

$$\frac{e(0)g'(h(0))}{v'(h(0))} = \int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda$$

Claim: This equation implies a negative relationship between $h(0)$ and λ^*.

Proof: Follows immediately.

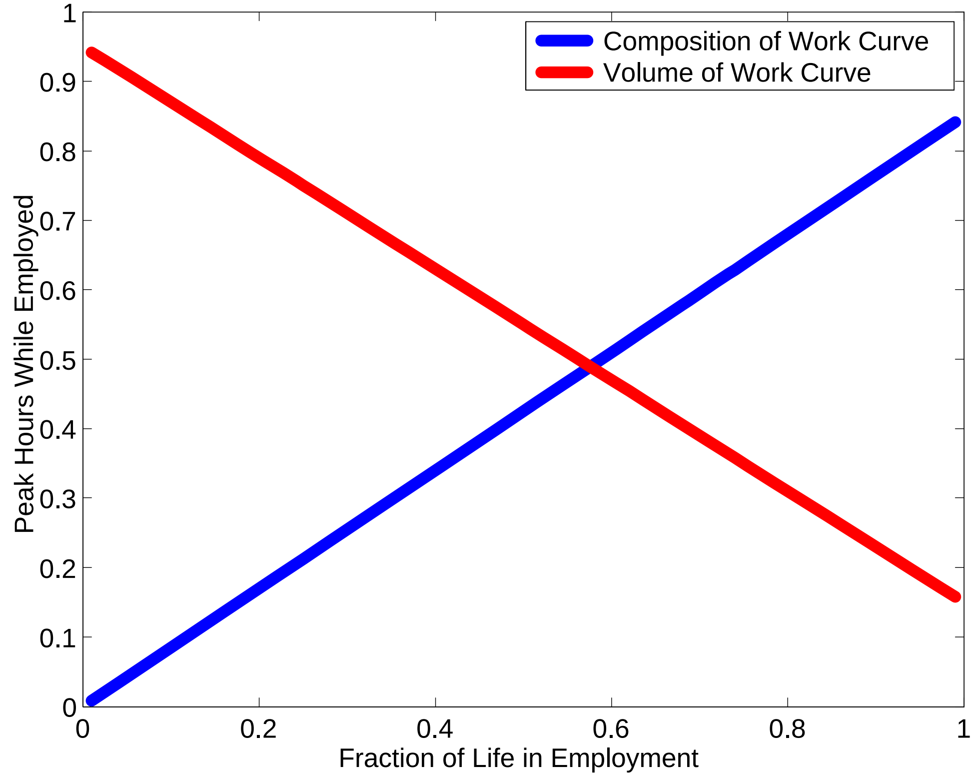
Summary:

Solving for steady state labor supply reduces to solving for $h(0)$ and λ^* from the following two equations:

$$\frac{v(h(\lambda^*))}{e(\lambda^*)g(h(\lambda^*))} = \frac{v'(h(0))}{e(0)g'(h(0))}$$

$$[\int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda]^{-1} = \frac{v'(h(0))}{e(0)g'(h(0))}$$

Graphical depiction of equilibrium labor supply:



Analysis of Tax/Transfer Policies

One can show that an increase in τ lowers the downward sloping curve but leaves the other curve unchanged.

It follows that an increase in τ leads to lower $h(0)$ and lower λ^*.

Quantitative Analysis

Functional Forms:

$$v(h) = \alpha \frac{h^{1+\gamma}}{1+\gamma}$$

$$g(h) = (h - \bar{h}) \text{ for } h \geq \bar{h}, \ 0 \text{ otherwise}$$

$$e(\lambda) = e_0 - (e_0 - e_1)\lambda.$$

Parameters to be selected: α, γ, $\bar{h}$, e_0 and e_1.

With log preferences, a proportional shift in $e(\lambda)$ has no effect on labor supply, so that we can normalize $e_1 = 1$ wlog.

Procedure for Selecting Parameters

We want to illustrate the effects of variation in γ on the model's properties, so I will consider various values of γ, in particular $\gamma = .5$, 1, 2, and 10. The tax rate τ is set to $.3$. For each value of γ, the other parameters are set so that:

- $\lambda^* = .67$
- $h(0) = .45$
- ratio of highest to lowest wages is 2

Remark about wages:

In the model, the wage per unit of labor services is constant at 1.

We define wages per unit of time in the model the same way as in the data:

$$w^h = earnings/hours\ worked$$

The target used in calibration is:

$$\frac{w^h(0)}{w^h(\lambda^*)} = \frac{e(0)g(h(0))/h(0)}{e(\lambda^*)g(h(\lambda^*))/h(\lambda^*)} = 2$$

Micro Labor Supply Elasticities

Dating back to MaCurdy (1981) there is a long tradition of using individual panel data to run regressions of the following form:

$$\log(h_{it}) = b_i + b_0 + b_1 \log(w_{it}^h) + \varepsilon_{it}$$

for a sample of prime-aged individuals.

We can generate such a sample in our model for each value of γ and run the above regression. We get the following results:

Table 1

Implied Frisch Elasticities

$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 10$
1.29	.59	.28	.05

Taxes and Hours of Work

We now ask what happens to the labor allocation if we raise τ from $.3$ to $.5$.

Table 2

Relative Outcomes for $\tau = .5$

γ	H	λ^*	$h(0)$
.50	.777	.857	.856
1.00	.784	.825	.918
2.00	.788	.808	.956
10.00	.790	.794	.991

Macro Labor Supply Elasticities

Consider an economist who wanted to interpret the steady state differences in aggregate hours shown previously using the following static labor supply model:

There is a single household, with preferences:

$$\log(c) - \mu \frac{h^{1+\theta}}{1+\theta}$$

Technology is given by:

$$c = h$$

Government: Taxes labor income at rate τ and uses the proceeds to fund a lump-sum transfer to the household.

Model implies:

$$h = [\frac{1-\tau}{\mu}]^{1/(\theta+1)}$$

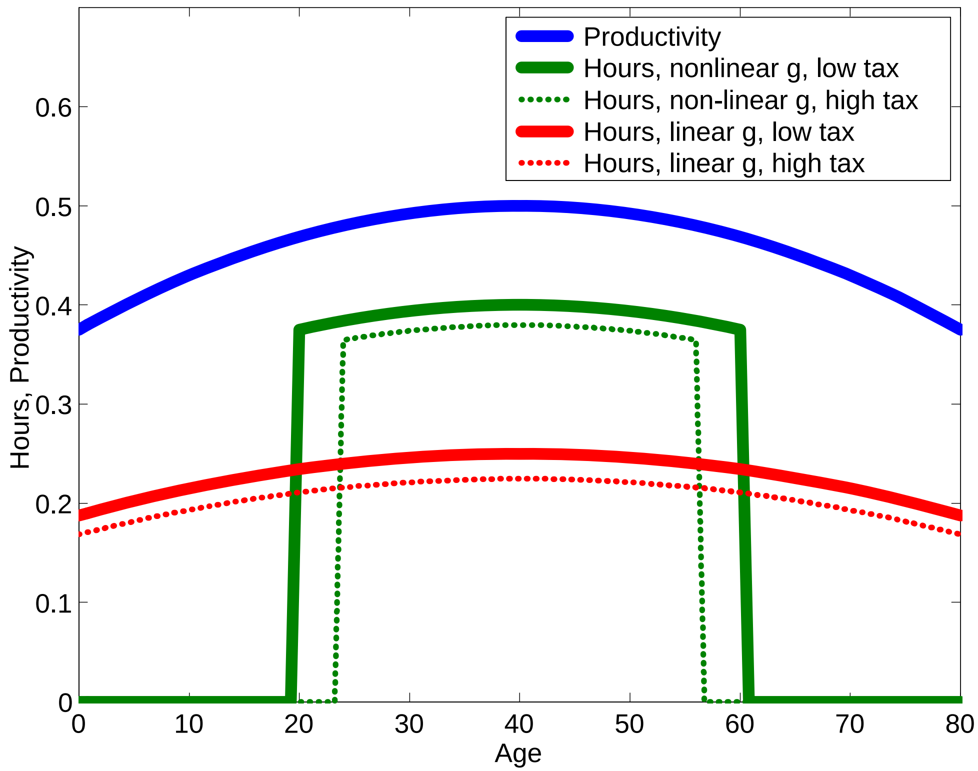
If we let h_i denote the hours that correspond to a country with tax rate τ_i, for $i = 1,2$, then using the above expression to interpret data data on taxes and hours of work leads to the following expression for θ :

$$\theta = \frac{\log(1-\tau_{1)} - \log(1-\tau_2)}{\log(h_1) - \log(h_2)} - 1$$

Table 3

Implied Values for θ

$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 10$
.33	.38	.41	.43


Productivity
Hours, nonlinear g, low tax
Hours, non-linear g, high tax
Hours, linear g, low tax
Hours, linear g, high tax
Hours, Productivity
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
80
Age

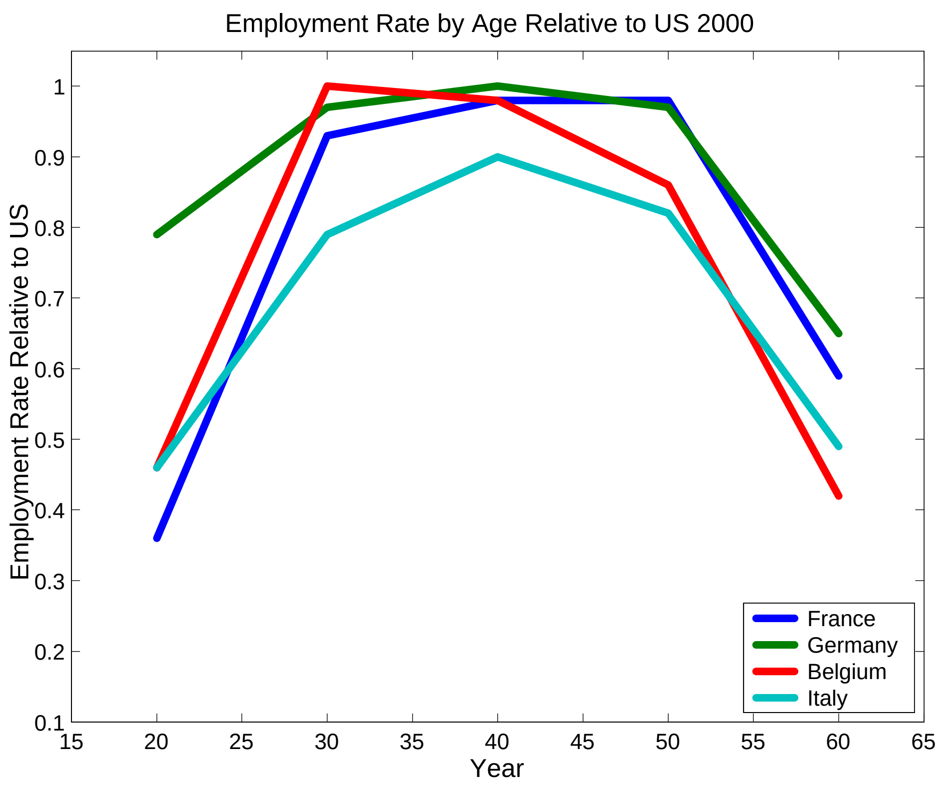

Employment Rate by Age Relative to US 2000
Employment Rate Relative to US
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
15
20
25
30
35
40
45
50
55
60
65
Year
France
Germany
Belgium
Italy

Table 6

Market Work in Europe Relative to the US				
	Belgium	France	Germany	Italy
Hours/Pop	.71	.68	.73	.69
Emp/Pop	.83	.88	.91	.79
Hours/Emp	.86	.77	.80	.87

Summary

- I have presented one explicit model in which:

  1. There is no inconsistency between small micro elasticities and large macro elasticities

  2. The micro elasticity is virtually irrelevant for the aggregate response

- Model does a good job of accounting for some patterns in cross-country data
- While these results are obtained in the context of a particular model, we believe that similar findings may obtain in other models that generate discontinuous changes in hours such as occurs at retirement.